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# On the Theory of Critical Currents and Flux Flow in Superconductors by the Mechanism of Plastic Deformation of the Flux-Line Lattice

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***Abstract*—In this paper I will discuss how the nature of the stress state in the flux-line lattice (FLL) of superconductors arises from the distribution, density, geometry, and strength of pinning centers. Under certain conditions this stress causes the onset of plastic deformation in the FLL for values of the current density below that required for flux-flow by general depinning. I will describe an analytic framework, based on a theory of plasticity of the FLL, which describes the flux-flow characteristics, including the possibility of thermally-activated flow and flux creep.**

## I. Introduction

It has been known since the classic work of E. J. Kramer [1] in the 1970s that for some superconductors the critical current density ($J_c$) is controlled by plastic shear of the flux-line lattice (FLL), rather than by a general, overall depinning instability. That is, flux-flow begins heterogeneously rather than homogeneously. This effect has been inferred from experimental data on the dependence of $J_c$ on the magnetic induction (B) [1] and has been illustrated by means of computer simulation [2]. Understanding of FLL plastic shear was developed further by Pruymboom et al. [3], who utilized as a measure of the strength of the FLL the theoretical shear strength of a perfect FLL, approximately $0.05C_{66}$, where $C_{66}$ is the FLL shear modulus. Pruymboom et al. recognized that the presence of dislocations in the FLL would reduce the strength of the FLL and hence lower the value of $J_c$, but did not explicitly develop this aspect of the problem. Later, Wördenweber and Abd-El-Hamed [4] utilized the concept of plastic flow of the FLL within weak "channels" in the pinning distribution to derive simulated voltage-current curves which are very similar to those-observed for $YBa_2Cu_3O_7$. In a subsequent short paper [5] I used the theory of dislocation dynamics and plasticity in metals to generalize and extend the earlier work of Pruymboom et al. [3] and Wördenweber and Abd-El-Hamed [4] to include a more realistic description of the effect of pinning strengths and densities on the plastic shear strength, as well as to include explicitly the effects of thermal activation.

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Recently, Tonomura et al. [6] used Lorentz electron microscopy to image a variety of types of plastic flow in FLLs within $Bi_2Sr_2CaCu_2O_8$ crystals. The type of plastic flow observed depends on the temperature, magnetic induction, and on the size and density of dominant pinning centers. In this paper I will sketch the outlines of a theoretical framework within which to examine the roles of such factors.

## II. Plastic Shear or General Depinning?

At a sufficiently large current density (J), the Lorentz forces on the flux lines will overcome the pinning forces and flux flow commences. If the pinning centers are uniform in strength and uniformly distributed and their density is comparable to the density of flux lines, then conditions are favorable for the onset of flux flow by homogeneous, wide-spread depinning at a large enough value of the current density. If, however there is a spatially-varying distribution in strength and/or density of pinning centers, then a state of non-hydrostatic stress arises in the FLL. If the shear components of this stress exceed the critical shear strength, i.e. the flow stress $\tau_c$, of the FLL, then inhomogeneous flux flow will occur by the mechanism of plastic deformation. A very simplified model, in which the distribution of pinning force density varies periodically with distance along a direction perpendicular to the direction of the Lorentz force on the flux lines, illustrates the issues involved. Suppose that the distribution consists of alternate slabs with relatively weak (w) and strong (s) pinning, of width $q\lambda$ and $(1-q)\ \lambda$ respectively; *i.e.*, q is the fraction of the FLL at the earliest risk of flow and $\Lambda$ is the wavelength. In such a case, the criterion that flux flow will occur by plastic shear is found to be [5]:

$$\tau_c < \left(B/2\varphi_o\right)\Lambda q\left(1-q\right)\left[\left(g f_p^m\right)_s - \left(g f_p^m\right)_w\right] \tag{1}$$

where B is the magnetic induction, $\varphi_o$ is the flux quantum, $f_p^m$ is the maximum value of the elementary pinning force per unit length of flux line, g is the probability that a given flux line is pinned, and the subscripts s and w denote the strong and weak slabs respectively. It can be seen that the occurrence of FLL plasticity depends on the flux-line density ($B/\varphi_o$) and the amplitude and wavelength of the variations of pinning force density, as well as the strength of the FLL resistance against plasticity ($\tau_c$).

## III. The Critical Current Density and Current-Voltage Relations

Assume that the strength and structure of pinning centers meets the criteria above, so that plastic shear of the FLL is the mechanism of flux motion. In order to derive the current-voltage (I-V) relations of the superconductor it is necessary to know the critical shear stress $\tau_c$ of the FLL, and this depends on, among other factors, the ratio of the pinning center density to the flux line density. When these densities are comparable, the FLL may not support the presence of dislocations, in which case the shear strength of the FLL is approximately that of a perfect lattice, $.05C_{66}$, see, e.g. [3], where $C_{66}$ is the temperature-and-magnetic-field-dependent elastic shear modulus of the FLL. In this case, the I-V curve can be derived from a step function (from dissipation-free behavior to flux-flow in channels between pinning centers) convoluted with a distribution function for channel widths [4].

For lower ratios of pin density to flux-line density, the FLL can sustain a population of dislocations, as shown experimentally by Tonomura et al. [6]. In this case, the FLL strain rate $\dot{\varepsilon}$, and hence the average flux-line velocity, which is proportional to the voltage V from (see, e.g. (9) in [5]), is described by the Orowan kinetic equation:

$$V \quad \dot{\varepsilon} = \rho_m v b \tag{2}$$

where $\rho_m$ is the density of mobile dislocations, b is the dislocation Burgers vector, given the present case by the FLL spacing $(\varphi_o/B)^{1/2}$, and v is average dislocation velocity arising from the acting stress. The latter factor depends upon whether or not the temperature is high enough for thermal activation to be significant. Neglecting thermal activation, the dislocation velocity v, in analogy with the case of metals and alloys, is expected to be proportional to $(\tau - \tau_c)^n$ where the exponent n is in the range 1-10 and $\tau_c$ rises monotonically with the dislocation density [7]. For the pinning center distribution described in section I, the stress on the deforming FLL is related to the local current density J by [5]:

$$\tau = 1/2 \quad \Lambda q B \left[ J - \left( g f_p^m / \varphi_o \right)_w \right] \quad . \tag{3}$$

With a suitable description of the FLL flow stress $\tau_c$, this yields the dislocation velocity and combining this with (2) yields the predicted I-V curve. The density of mobile dislocations, $\rho_m$, must be specified. Not much is known about this for the FLL, but it probably increases with increasing velocity v. (The derivation of the I-V curve when thermal activation is significant is discussed in [5] and will not be discussed further here for lack of space.)

## IV. Conclusions

By analogy with the physics of dislocation dynamics and plasticity in metals and alloys, a theoretical framework can be derived to describe current-voltage relations for superconductors in which flux flow arises from the plastic deformation of the flux-line lattice.

## References


[1] E. J. Kramer, *"Scaling Laws for Flux Pinning in Hard Superconductors," J. Appl. Phys.*, vol. 44, pp. 1360-1370, March 1973.

[2] C. J. Olson, C. Reichardt, and F. Nori, *"Superconducting Vortex Avalanches, Voltage Bursts, and Vortex Plastic Flow: Effect of the Microscopic Pinning Landscape on the Macroscopic Properties," Phys. Rev. V.*, vol. 56, pp. 6175-6194, 1 September 1997-II.

[3] A. Pruymboom, P. H. Kes, E. van der Drift, and s. Radelaar, *"Shear-limited Flux Pinning in superconducting Thin-film Devices," Appl. Phys. Lett.*, vol. 52, pp. 662-664, February 1988.

[4] R. Wördenweber and M. O. Abd-El-Hamed, *"Interpretation of I-V Characteristics Measured on Epitaxially Grown Thin Films of $YBa_2Cu_3O_7$," Supercond. Sci. Technol.,"* vol. 5, pp. S113-S116, January 1992..

[5] D. O. Welch, *"Concerning Factors Which Determine Whether Flux-Lattice Shear or Pin Breaking Limits the Critical Current-Density of Superconductors," IEEE Trans. Appl. Supercond.*, vol. 3, pp. 1476-1478, March 1993.

[6] A. Tonomura, H. Kasai, O. Kamimura, T. Matsuda, K. Harada, J. Shimoyama, K. Kishio, and K. Kitazawa, *"Motion of Vortices in Superconductors," Nature*, vol. 397, pp. 308-309, 28 January 1999.

[7] T. Suzuki, S. Takeuchi, and H. Yoshinaga, in *Dislocation Dynamics and Plasticity*, Springer-Verlag, 1991.